International Journal of Theoretical Physics, Vol. 43, No. 12, December 2004 (© 2004)

Hamiltonian and Path Integral Formulations of the Nambu-Goto D1-Brane Action With and Without a Dilaton Field Under Gauge-Fixing

Usha Kulshreshtha[1] and D. S. Kulshreshtha[2,3]

The Hamiltonian and path integral formulations of the Nambu-Goto D1-brane action with and without a scalar dilaton field are investigated under appropriate gauge-fixing.

KEY WORDS: Nambu-Goto action; D1-brane; string theory.

1. INTRODUCTION

The Nambu-Goto action is a very important and widely studied action in string theories (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). In the present work, we study the Hamiltonian and path integral formulations (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976) of this action (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996) describing the D1-brane with and without a scalar dilaton field φ under appropriate gauge-fixing conditions (GFC's).

In the next section, the action is considered without the scalar dilaton field and in Section 3, the action is studied in the presence of the scalar dilaton field. The Hamiltonian and path integral quantizations are studied in both the cases under appropriate canonical GFC's in the absence of boundary conditions (BCs). Finally the summary and discussion is presented in Section 4.

[1] Department of Physics, Keshav Mahavidyalaya, University of Delhi, Keshavpuram, Delhi, India.
[2] Department of Physics and Astrophysics, University of Delhi, Delhi, India.
[3] To whom correspondence should be addressed at Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India; e-mail: dsk@physics.du.ac.in.; dsk@physik.uni-kl.de.

0020-7748/04/1200-2355/0

2. THE ACTION WITHOUT A SCALAR DILATON FIELD

The Nambu-Goto action describing the propagation of a D1-brane in a d-dimensional flat background (with $d = 10$ for the fermionic and $d = 26$ for bosonic D1-brane) is defined by (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996):

$$S_1 = \int \mathcal{L}_1 d^2\sigma \tag{1}$$

$$\mathcal{L}_1 = [-T]\left[[-\det(G_{\alpha\beta})]^{\frac{1}{2}}\right] \tag{2}$$

$$\mathcal{L}_1 = \left[-T[-\det(\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu})]^{\frac{1}{2}}\right] \tag{3}$$

$$\mathcal{L}_1 = \left[-T[(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2]^{\frac{1}{2}}\right] \tag{4}$$

$$\mathcal{L}_1 = [-\mathrm{TL}] \tag{5}$$

$$G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}; \quad \eta_{\mu\nu} = \mathrm{diag}(-1, +1, \ldots, +1) \tag{6}$$

$$\mu, \nu = 0, 1, \ldots, (d-1); \quad \alpha, \beta = 0, 1 \tag{7}$$

$$L^2 = [(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2] \tag{8}$$

$$\dot{X}^\mu \equiv \frac{\partial \mathrm{X}^\mu}{\partial\tau}, \quad X'^\mu \equiv \frac{\partial \mathrm{X}^\mu}{\partial\sigma} \tag{9}$$

In the present work we would consider only the bosonic D1 brane (sometimes also called the D-string) with $d = 26$ (however, for the corresponding fermionic case one has $d = 10$). Here $\sigma^\alpha \equiv (\tau, \sigma)$ are the two parameters describing the world-sheet (WS). The overdots and primes denote in general, the derivatives with respect to the WS coordinates τ and σ. The string tension T is a constant of mass dimension two. $G_{\alpha\beta}$ is the induced metric on the WS and $X^\mu(\tau, \sigma)$ are the maps of the WS into the d-dimensional Minkowski space and describe the strings evolution in space time (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

Further the theory described by the action S_1 is a gauge-invariant (GI) (and consequently a gauge nonanomalous) theory possessing the usual three local gauge symmetries given by the two-dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

The canonical momenta obtained from (1) are

$$\Pi^{\mu} := \frac{\partial \mathcal{L}_1}{\partial(\partial_\tau X_\mu)} = \left[\frac{-T}{L}\right][(\dot{X}\cdot X')X'^{\mu} - (X')^2\dot{X}^{\mu}] \tag{10}$$

$$\partial_\tau \equiv \frac{\partial}{\partial\tau}, \quad \partial_\sigma \equiv \frac{\partial}{\partial\tau} \tag{11}$$

where Π^{μ} are the canonical momenta conjugate respectively to X_μ. The theory described by S_1 is thus seen to possess two primary constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$\Psi_1 = (\Pi\cdot X') \approx 0 \tag{12}$$

$$\Psi_2 = [\Pi^2 + T^2(X')^2] \approx 0 \tag{13}$$

Here the symbol $\approx$ denotes a weak equality (WE) in the sense of Dirac (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976), and it implies that these above constraints hold as strong equalities only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality (WOE) for the corresponding quantum theory) (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976). The constraints Ψ_1 and Ψ_2 expressed by (3) are, infact, the usual so-called Virasoro constraints of the theory (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). The canonical Hamiltonian density corresponding to $\mathcal{L}_1$ is:

$$\mathcal{H}_1^c = [\Pi^{\mu}(\partial_\tau X_\mu) - \mathcal{L}_1] = 0 \tag{14}$$

It is seen to vanish identically using (1) and (2), the dynamics of the system is thus completely determined by the constraints of the theory. After incorporating the primary constraints of the theory in the canonical Hamiltonian density $\mathcal{H}_1^c$ with the help of Lagrange multiplier fields $u_1(\tau,\sigma)$ and $u_2(\tau,\sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written as:

$$\mathcal{H}_1^T = [u_1\Psi_1 + u_2\Psi_2] \tag{15}$$

$$\mathcal{H}_1^T = [u_1(\Pi\cdot X') + u_2[\Pi^2 + T^2(X')^2]] \tag{16}$$

We denote the momenta canonically conjugate to u_1 and u_2 by p_{u_1} and p_{u_2}, respectively. The Hamiltons equations of motion obtained from the total

Hamiltonian

$$H_1^T = \mathcal{H}_1^T d\sigma \tag{17}$$

e.g., for the closed string with periodic boundary conditions (BC's) (with $\partial_\tau \equiv \frac{\partial}{\partial\tau}$ and $\partial_\sigma \equiv \frac{\partial}{\partial\sigma}$) are

$$\partial_\tau X^\mu = \frac{\partial H_1^T}{\partial \Pi_\mu} = [u_1 X'^\mu + 2u_2 \Pi^\mu] \tag{18}$$

$$-\partial_\tau \Pi^\mu = \frac{\partial H_1^T}{\partial X_\mu} = -\partial_\sigma [u_1 \Pi^\mu + 2X'^\mu T^2 u_2] \tag{19}$$

$$\partial_\tau u_1 = \frac{\partial H_1^T}{\partial p_{u_1}} = 0 \tag{20}$$

$$-\partial_\tau p_{u_1} = \frac{\partial H_1^T}{\partial u_1} = (\Pi \cdot X') \tag{21}$$

$$\partial_\tau u_2 = \frac{\partial H_1^T}{\partial p_{u_2}} = 0 \tag{22}$$

$$-\partial_\tau p_{u_2} = \frac{\partial H_1^T}{\partial u_2} = [\Pi^2 + T^2 (X')^2] \tag{23}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraints Ψ_1 and Ψ_2 be preserved in the course of time one does not get any further constraints. The theory is thus seen to posses only two constraints Ψ_1 and Ψ_2. The first-order Lagrangian density of the theory is

$$\mathcal{L}_1^{\mathrm{IO}} = [\Pi^\mu(\partial_\tau X_\mu) + p_{u_1}(\partial_\tau u_1) + p_{u_2}(\partial_\tau u_2) - \mathcal{H}_1^T] \tag{24}$$

$$\mathcal{L}_1^{\mathrm{IO}} = [[\Pi^2 + T^2 (X')^2] u_3] \tag{25}$$

The matrix of Poission brackets of the constraints Ψ_i is seen to be a singular matrix implying that the set of constraints Ψ_i is first-class (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976) and that the theory described by S_1 is a gauge invariant (GI) theory (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976). It is rather well known that the theory described by S_1 indeed possesses three local gauge symmetries given by the two dimensional WS reparametrization invariance (WSRI) and the Weyl invariance (WI) (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996). To study the Hamiltonian and path integral formulations of the theory under gauge fixing we convert the set of first-class constraints Ψ_i into a set of second-class constraints by imposing arbitrarily, some additional constraints on the system called the gauge-fixing conditions(GFCS) or the gauge constraints. For this purpose, we could choose, e.g., the set of GFCS (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$\zeta_1 = X^2 \approx 0 \tag{26}$$

$$\zeta_2 = \Pi' \approx 0 \tag{27}$$

corresponding to this choice of GFCS, the total set of constraints of the theory under which the quantization of the theory could e.g., be studied becomes

$$\Psi_1 = (\Pi \cdot X') \approx 0 \tag{28}$$

$$\Psi_2 = [\Pi^2 + T^2(X')^2] \approx 0 \tag{29}$$

$$\Psi_3 = \zeta_1 = X^2 \approx 0 \tag{30}$$

$$\Psi_4 = \zeta_2 = \Pi' \approx 0 \tag{31}$$

We now calculate the matrix of $M_{\alpha\beta}(:= \{\Psi_\alpha, \Psi_\beta\}_{\text{PB}})$ of the Poisson brackets of the constraints Ψ_i. The nonvanishing elements of the matrix $M_{\alpha\beta}$ are obtained as

$$M_{13} = -M_{31} = [-2X']\delta(\sigma - \sigma') \tag{32}$$

$$M_{14} = -M_{41} = [-\Pi]\delta''(\sigma - \sigma') \tag{33}$$

$$M_{23} = -M_{32} = [-4\Pi]\delta(\sigma - \sigma') \tag{34}$$

$$M_{24} = -M_{42} = [-2X'T^2]\delta''(\sigma - \sigma') \tag{35}$$

The matrix $M_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of constraints Ψ_i is a set of second-class constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976). The determinant of the matrix $M_{\alpha\beta}$ is given by

$$[\|\det(M_{\alpha\beta})\|]^{1/2} = 4M\delta''(\sigma - \sigma')\delta(\sigma - \sigma') \tag{36}$$

$$M = [\Pi^2 - T^2(X'^2] \tag{37}$$

The nonvanishing elements of the inverse of the matrix $M_{\alpha\beta}$ (i.e., the elements of the matrix $(M^{-1})_{\alpha\beta}$) are obtained as

$$(M^{-1})_{13} = -(M^{-1})_{31} = \left[\frac{-T^2(X')}{2M}\right]\delta(\sigma - \sigma') \tag{38}$$

$$(M^{-1})_{14} = -(M^{-1})_{41} = \delta(\sigma - \sigma') \tag{39}$$

$$(M^{-1})_{23} = -(M^{-1})_{32} = \left[\frac{\Pi}{(2M)}\right]|\sigma - \sigma'| \tag{40}$$

$$(M^{-1})_{14} = -(M^{-1})_{41} = \left[\frac{\Pi}{(4M)}\right]\delta(\sigma - \sigma') \tag{41}$$

$$(M^{-1})_{24} = -(M^{-1})_{42} = \left[\frac{-X'}{(4M)}\right]|\sigma - \sigma'| \tag{42}$$

with

$$\int M(\sigma, \sigma'')M^{-1}(\sigma'', \sigma')d\sigma'' = \mathbf{1}_{4\times 4}\delta(\sigma - \sigma') \tag{43}$$

Now following the standard Dirac quantization procedure in the Hamiltonian formulation (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c, d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976), the noinvanishing equal WS time (EWST) Dirac brackets (DB's) (denoted by $\{\, ,\ \}_D$) of the theory described by the action S_1 under the GFC's ζ_i are obtained (with the arguments of the variables being suppressed) as (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$\{X^\mu(\sigma, \tau), \Pi_\nu(\sigma', \tau)\}_{\mathrm{DB}} = (-i)\delta^\mu{}_\nu\delta(\sigma - \sigma') \tag{44}$$

It is important to recall here that the constraints of the theory represent only the weak equalities in the sense of Dirac (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976), as explained in the foregoing implying that they are strongly zero only on the reduced hypersurface of the constraints and not in the rest of the phase space of the (classical) theory (with a similar weak operator equality holding for the corresponding quantum theory).

Further, in the canonical quantization of the theory while going from equal WS time (EWST) Dirac brackets of the theory to the corresponding EWST commutation relations one would encounter here the problem of operator ordering (Chu and Ho, 2000; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976) because the product of canonical variables of the theory are involved in the classical description of the theory (like in the expressions for the constraints of the theory) as well as in the calculation of the Dirac brackets. These variables are envisaged as noncommuting operators in the quantized theory leading to the problem of so-called operator ordering

(Chu and Ho, 2000; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976). This problem could, however, be resolved (Chu and Ho, 2000; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993;a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976) by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the Hermiticity of these operators (Chu and Ho, 2000; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976).

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z_1[J_i]$ of the theory under GFC's ζ_i in the presence of the external sources J_i (following the Senjanovic procedure Kulshreshtha and Kulshreshtha, 2003a,b,c for a theory possessing a set of second-class constraints, appropriate for our theory described by the action S_1 considered under the GFC's: ζ_i(12) Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976) as follows:

In the path integral formulation, the transition to the quantum theory, is, however, made by writing the vacuum to vacuum transition amplitude called the generating functional $Z_1[J_i]$ of the theory under GFC's ζ_i in the presence of external sources J_i as follows (Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993,a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$Z_1[J_i] := \int [d\mu] \exp\left[i \int d^2\sigma \left[\mathcal{L}_1^{\mathrm{IO}} + J_i \Phi^i \right] \right] \tag{45}$$

where the phase space variables of the theory are $\Phi^i \equiv (X^\mu, u_1, u_2)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_\mu, p_{u_1}, p_{u_2})$. The functional measure $[d\mu]$ of the generating functional $Z_1[J^i]$ under the GFCS ζ_i is obtained using Eqs. (8), (10), (12), and (16) as (Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$\begin{aligned}[d\mu] = {} & [4M\delta''(\sigma - \sigma')\delta(\sigma - \sigma')][dX^\mu][du_1][du_2][d\Pi_\mu][dp_{u_1}][dp_{u_2}] \\ & \delta[(\Pi.X') \approx 0]\delta[(\Pi^2 + T^2(X')^2) \approx 0]\delta[(X^2) \approx 0]\delta[(\Pi') \approx 0].\end{aligned} \tag{46}$$

The Hamiltonian and path integral quantization of our theory described by the action S_1 under GFCS ζ_i is now complete. In the next section we study this theory in the presence of the scalar dilation field φ.

3. THE ACTION IN THE PRESENCE OF A SCALAR DILATION FIELD

The (bosonic) Nambu-Goto action describing the propagation of a D1-brane in a d-dimensional flat background in the presence of a scalar dilation field φ is defined by (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996):

$$S_2 = \int \mathcal{L}_2 d^2\sigma \tag{47}$$

$$\mathcal{L}_2 = [e^{-\varphi}\mathcal{L}_1] \tag{48}$$

$$\mathcal{L}_2 = \left[-Te^{-\varphi}[(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2]^{\frac{1}{2}}\right] \tag{49}$$

$$\mathcal{L}_2 = [-Te^{-\varphi}L] \tag{50}$$

The momenta Π^μ, and π canonically conjugate respectively to X_μ and φ obtained from $\mathcal{L}_2$ are

$$\Pi^\mu := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau X^\mu)} = \left[\frac{-\mathrm{Te}^{-\varphi}}{L}\right][(\dot{X}\cdot X')X'^\mu - (X')^2\dot{X}^\mu] \tag{51}$$

$$\pi := \frac{\partial\mathcal{L}_2}{\partial(\partial_\tau\varphi)} = 0 \tag{52}$$

The theory described by S_2 is thus seen to posses three primary constraints:

$$\chi_1 = \pi \approx 0 \tag{53}$$

$$\chi_2 = (\Pi\cdot X') \approx 0 \tag{54}$$

$$\chi_3 = [\Pi^2 + T^2e^{-2\varphi}(X')^2] \approx 0 \tag{55}$$

The above constraints are again the usual Virasoro constraints of the theory in the presence of the dilaton field φ (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

The canonical Hamiltonian density corresponding to $\mathcal{L}_2$ is

$$\mathcal{H}_2^c = \left[\Pi^\mu(\partial_\tau X_\mu) + \pi(\partial_\tau\varphi) - \mathcal{L}_2\right] = 0 \tag{56}$$

It is seen to vanish identically using (18) and (19). The dynamics of the system is thus (again) completely determined by the constraints of the theory. After incorporating the primary constraints of the theory in the canonical Hamiltonian density $\mathcal{H}_2^c$ with the help of Lagrange multiplier fields $v_1(\tau,\sigma)$, $v_2(\tau,\sigma)$ and $v_3(\tau,\sigma)$, which we treat as dynamical, the total Hamiltonian density of the theory could be written as

$$\mathcal{H}_2^T = \left[\mathcal{H}_2^c + v_1\chi_1 + v_2\chi_2 + v_3\chi_3\right] \tag{57}$$

We denote the momenta canonically conjugate to v_1, v_2 and v_3, respectively by p_{v_1}, p_{v_2} and p_{v_3}. The Hamiltons equations obtained from the total Hamiltonian

$$H_2^T = \int \mathcal{H}_2^T \, d\sigma \tag{58}$$

e.g., for the closed string with periodic boundary conditions (BC's) are

$$\partial_\tau X^\mu = \frac{\partial H_2^T}{\partial \Pi_\mu} = [v_2 X\prime\mu + 2v_3\Pi^\mu] \tag{59}$$

$$-\partial_\tau \Pi^\mu = \frac{\partial H_2^T}{\partial X_\mu} = -\partial_\sigma[v_2\Pi^\mu + 2X'^\mu T^2 e^{-2\varphi} v_3] \tag{60}$$

$$\partial_\tau \varphi = \frac{\partial H_2^T}{\partial \pi} = v_1 \tag{61}$$

$$-\partial_\tau \pi = \frac{\partial H_2^T}{\partial \varphi} = [-2T^2 e^{-2\varphi}(X')^2 v_3] \tag{62}$$

$$\partial_\tau v_1 = \frac{\partial H_2^T}{\partial p_{v_1}} = 0 \tag{63}$$

$$-\partial_\tau p_{v_1} = \frac{\partial H_2^T}{\partial v_1} = \pi \tag{64}$$

$$\partial_\tau v_2 = \frac{\partial H_2^T}{\partial p_{v_2}} = 0 \tag{65}$$

$$-\partial_\tau p_{v_2} = \frac{\partial H_2^T}{\partial v_2} = (\Pi \cdot X') \tag{66}$$

$$\partial_\tau v_3 = \frac{\partial H_2^T}{\partial p_{v_3}} = 0 \tag{67}$$

$$-\partial_\tau p_{v_3} = \frac{\partial H_2^T}{\partial v_3} = [\Pi^2 + T^2 e^{-2\varphi}(X')^2] \tag{68}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. The preservation of χ_1, χ_2, and χ_3 in the course of time does not yield any further constraints, the theory is thus seen to possess only three constraints χ_1, χ_2, and χ_3. The first-order Lagrangian density of the theory is

$$\mathcal{L}_2^{\text{IO}} = \left[\Pi^\mu(\partial_\tau X_\mu) + \pi(\partial_\tau \varphi) + p_{v_1}(\partial_\tau v_1) + p_{v_2}(\partial_\tau v_2) + p_{v_3}(\partial_\tau v_3) - \mathcal{H}_2^T\right] \tag{69}$$

$$\mathcal{L}_2^{\text{IO}} = \left[[\Pi^2 + T^2 e^{-2\varphi}(X')^2]v_3\right] \tag{70}$$

The matrix of the Poisson brackets of the constraints χ_i is seen to be a singular matrix implying that the set of constraints χ_i is first-class (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976) and that the theory described by S_2 is a gauge-invariant (GI) theory. It is rather well known that the theory described by S_2 indeed posseses three local gauge symmetries given by the two-dimensioanl WS reparametrization invariance (WSRI) and teh Weyl invariance (WI) (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996).

To study the Hamiltonian and path integral formulations of this GI theory under GFC's, we convert the set of first-class constraints of the theory χ_i into a set of second-class constraints, by imposing arbitrarily, some additional constraints on the system called the GFC's or the gauge constraints. For this purpose, we could choose, for example, the set of GFC's (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$\rho_1 = X^2 \approx 0 \tag{71}$$

$$\rho_2 = \Pi' \approx 0 \tag{72}$$

$$\rho_3 = \varphi \approx 0 \tag{73}$$

corresponding to this choice of GFCS, the total set of constraints of the theory under which the quantization of the theory could e.g., be studied becomes

$$\chi_1 = \pi \approx 0 \tag{74}$$

$$\chi_2 = (\Pi \cdot X') \approx 0 \tag{75}$$

$$\chi_3 = [\Pi^2 + T^2 e^{-2\varphi}(X')^2] \approx 0 \tag{76}$$

$$\chi_4 = \rho_1 = X^2 \approx 0 \tag{77}$$

$$\chi_5 = \rho_2 = \Pi' \approx 0 \tag{78}$$

$$\chi_6 = \rho_4 = \varphi \approx 0 \tag{79}$$

We now calculate the matrix of $R_{\alpha\beta}(:= \{\chi_\alpha, \chi_\beta\}_{\text{PB}})$ of the Poisson brackets of the constraints χ_i. The non vanishing elements of the matrix $R_{\alpha\beta}$ are obtained as

$$R_{13} = -R_{31} = [2T^2 e^{-2\varphi} X'^2]\delta(\sigma - \sigma') \tag{80}$$

$$R_{16} = -R_{61} = [-1]\delta(\sigma - \sigma') \tag{81}$$

$$R_{24} = -R_{42} = [-2X']\delta(\sigma - \sigma') \tag{82}$$

$$R_{25} = -R_{52} = [-\Pi]\delta''(\sigma - \sigma') \tag{83}$$

$$R_{34} = -R_{43} = [-4\Pi]\delta(\sigma - \sigma') \tag{84}$$

$$R_{35} = -R_{53} = [-2X'T^2 e^{-2\varphi}]\delta''(\sigma - \sigma') \tag{85}$$

The matrix $R_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of constraints χ_i is a set of second-class constraints (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976). The determinant of the matrix $R_{\alpha\beta}$ is given by

$$[\| \det(R_{\alpha\beta})\|]^{1/2} = [4R\delta''(\sigma - \sigma')\delta^2(\sigma - \sigma')] \tag{86}$$

$$R = [\Pi^2 - T^2 e^{-2\varphi}(X'^2)] \tag{87}$$

and the nonvanishing elements of the inverse of the matrix $R_{\alpha\beta}$ (i.e., the elements of the matrix $(R^{-1})_{\alpha\beta}$) are obtained as

$$(R^{-1})_{16} = -(R^{-1})_{61} = \delta(\sigma - \sigma') \tag{88}$$

$$(R^{-1})_{24} = -(R^{-1})_{42} = \left[\frac{-T^2 e^{-2\varphi}(X')}{2R}\right]\delta(\sigma - \sigma') \tag{89}$$

$$(R^{-1})_{25} = -(R^{-1})_{52} = \left[\frac{\Pi}{(2R}\right]|\sigma - \sigma'| \tag{90}$$

$$(R^{-1})_{34} = -(R^{-1})_{43} = \left[\frac{\Pi}{(4R)}\right]\delta(\sigma - \sigma') \tag{91}$$

$$(R^{-1})_{35} = -(R^{-1})_{53} = \left[\frac{-X'}{(4R)}\right]|\sigma - \sigma'| \tag{92}$$

$$(R^{-1})_{46} = -(R^{-1})_{64} = \left[\frac{-T^2 e^{-2\varphi}(X'^2)\Pi}{2R}\right]\delta(\sigma - \sigma') \tag{93}$$

$$(R^{-1})_{56} = -(R^{-1})_{65} = \left[\frac{T^2 e^{-2\varphi}(X'^2)(X')}{2R}\right]|\sigma - \sigma'| \tag{94}$$

with

$$\int R(\sigma, \sigma'')R^{-1}(\sigma'', \sigma')d\sigma'' = \mathbf{1}_{6\times 6}\delta(\sigma - \sigma') \tag{95}$$

Now following the standard Dirac quantization procedure (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976) in the Hamiltonian formulation (Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b,; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976), the nonvanishing EWST Dirac brackets (denoted by $\{\,,\,\}_D$) of the theory in the presence of a scalar dilation field described by the action S_2 under the GFC's ρ_i are obtained (with the arguments of the field variables being supperesed as Dirac, 1950; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a, b,c; Senjanovic, 1976):

$$\{X^\mu(\sigma, \tau), \Pi_\nu(\sigma', \tau)\}_{\text{DB}} = (-i)\delta^\mu_\nu\delta(\sigma - \sigma') \tag{96}$$

$$\begin{aligned}\{X^\mu(\sigma, \tau), \pi(\sigma', \tau)\}_{\text{DB}} &= [[1/(2R)][X' - (X')^2][T^2e^{-2\varphi}(X')^2].\\ &\quad\times(X^\mu)\epsilon(\sigma - \sigma')]\end{aligned} \tag{97}$$

$$\begin{aligned}\{\pi(\sigma, \tau), \Pi^\mu(\sigma', \tau)\}_{\text{DB}} &= [[1/(2R)][X' - (X')^2][T^2e^{-2\varphi}(X')^2)].\\ &\quad\times(\Pi'^\mu)|\sigma - \sigma'|]\end{aligned} \tag{98}$$

As explained in the previous section, the nonvanishing DB's involving the gauge field A_1, in the above results, would become strongly zero on the reduced hypersurface of the constraints of the theory described by the action S_2 (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Gitman and Tyutin, 1990; Johnson, 2000; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Senjanovic, 1976; Tseytlin, 1996). The problem of operator ordering occurring here while making a transition from the EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Section 3, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators (Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976).

In the path integral formulation, the transition to quantum theory is made again by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z_2[J_i]$ of the theory, following again the Senjanovic procedure for a theory possessing a set of second-class constraints (Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976), appropriate for our theory described by the action S_2 considered under the GFC's ρ_i, in the presence of the external sources J_i as follows (Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Senjanovic, 1976):

$$Z_2[J_i] := \int [d\mu] \exp\left[i \int d^2\sigma \left[\mathcal{L}_2^{\mathrm{IO}} + J_i \Phi^i\right]\right] \tag{99}$$

where the phase space variables of the theory are $\Phi^i \equiv (X^\mu, \varphi, v_1, v_2, v_3)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_\mu, \pi, p_{v_1}, p_{v_2}, p_{v_3})$. The functional measure $[d\mu]$ of the generating functional $Z_2[J_i]$ under the GFCS ρ_i is obtained using Eqs. (25), (27), (29), and (33) as (Kulshreshtha and Kulshreshtha, 2003a,b,c):

$$\begin{aligned}[d\mu] = {} & [4R\delta''(\sigma - \sigma')\delta^2(\sigma - \sigma')][dX^\mu][d\varphi][dv_1][dv_2][dv_3] \\ & \times [d\Pi_\mu][d\pi][dp_{v_1}][dp_{v_2}][dp_{v_3}]\delta[(\pi) \approx 0]\delta[(\Pi.X') \approx 0] \\ & \times \delta[(\Pi^2 + T^2 e^{-2\varphi}(X')^2) \approx 0]\delta[(X^2) \approx 0]\delta[(\Pi') \approx 0]\delta[(\varphi) \approx 0].\end{aligned} \tag{100}$$

The Hamiltonian and path integral quantization of the theory described by the action S_2 under the GFC's ρ_i is now complete.

4. SUMMARY AND DISCUSSION

In this work we have studied the Hamiltonian and path integral quantization of a Nambu-Goto action (Luest and Theisen, 1989; Brink and Henneaux, 1988; Johnson, 2000; Aganagic *et al.*, 1997; Abou Zeid and Hull, 1997; Schmidhuber, 1996; de Alwis and Maharana, 2000; Mukhi, 1997; Sato, 1996; Tseytlin, 1996). We have studied this action describing the D1-brane action with and without a scalar dilaton field φ, under appropriate GFCS, in the absence of BCS, using the instant-form of dynamics on the hyperplanes of the WS defined by the hyperplanes: WS-time $= \sigma^0 = \tau =$ constant. The problem of operator ordering occurring here while making a transition from EWST Dirac brackets to the corresponding EWST commutation relations can be resolved here as explained in Section 2, by demanding that all the string fields and momenta of the theory are Hermitian operators and that all the canonical commutation relations be consistent with the hermiticity of these operators (Chu and Ho, 2000; Gitman and Tyutin, 1990; Kulshreshtha *et al.*, 1993a,b,c,d,e, 1994a,b; Kulshreshtha and Kulshreshtha, 2002a,b; Kulshreshtha and Kulshreshtha, 2003a,b,c; Maharana, 1983; Senjanovic, 1976).

It is important to mention here in our work we have not imposed any boundary conditions (BC's) for the open and closed strings separately. There are two ways to take these BC's into account: (a) one way is to impose them directly in the usual way for the open and closed strings separately in an appropriate manner (Abou Zeid and Hull, 1997; Aganagic *et al.*, 1997; Brink and Henneaux, 1988; de Alwis and Sato, 1996; Johnson, 2000; Kulshreshtha and Kulshreshtha, 2003a,b,c; Luest and Theisen, 1989; Maharana, 2000; Mukhi, 1997; Schmidhuber, 1996; Tseytlin, 1996), (b) an alternative second way is to treat these BC's as the Dirac primary constraints (Chu and Ho, 2000; Sheikh-Jabbari and Shirzad, 1999) and study the theory accordingly (Chu and Ho, 2000; Sheikh-Jabbari and Shirzad, 1999). At present our related work is underway and would be reported later.

REFERENCES

Abou Zeid, M. and Hull, C. M. (1997). *Intrinsic Geometry of D-Branes*. (hep-th/9704021).

Aganagic, M., Park, J., Popescu, C., and Schwarz, J. (1997). *Dual D-Brane Actions*. (hep-th/9702133).

Brink, L. and Henneaux, M. (1988). *Principles of String Theory*, Plenum, New York.

Chu, C. S. and Ho, P. M. (2000). Constrained quantization of open string in background B field and noncommutative D-brane. *Nuclear Physics B*, **568**, 447. [arXiv: hep-th/ 9906192] (CITATION = HEP-TH 9906192).

de Alwis, S. P. and Sato, K. (1996). *Physical Review D*, **53**, 7187–7196.

Dirac, P. A. M. (1950). *Canadian Journal of Mathematics* **2**, 129.

Gitman, D. M. and Tyutin, I. V. (1990). *Quantization of Fields with constraints*, Springer Verlag, Berlin.

Johnson, C. V. (2000). *D-Brane Primer*. hep-th/0007170.

Kulshreshtha, U. and Kulshreshtha, D. S. (2002a). *International Journal of Theoritical Physics* **41**, 1961–1976.

Kulshreshtha, U. and Kulshreshtha, D. S. (2002b). *International Journal of Theoritical Physics* **41**, 2411–2426.

Kulshreshtha, U. and Kulshreshtha, D. S. (2003a). *Physics Letters B* **555**(3/4), 255–263.

Kulshreshtha, U. and Kulshreshtha, D. S. (2003b). *European Physical Journal C* **29**, 453–461

Kulshreshtha, U. and Kulshreshtha, D. S. (2003c). University of Kaiserslautern. Preprints.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993a). (Invited talk by D.S. Kulshreshtha). In *Proceedings of the IInd Workshop on Constraint Theory and Quantization Methods*, Montepulciano (Siena), Italy, June 30–July 1, 1993.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993b). *Zeitschrift fur Physik C* **60**, 427–431.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993c). *Helvetica Physica Acta* **66**, 737–751.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993d). *Helvetica Physica Acta* **66**, 752–794.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1993e). *Physical Review D* **47**, 4634–4638.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1994a). World Scientific, Singapore, pp. 305–327.

Kulshreshtha, U., Kulshreshtha, D. S., and Mueller-Kirsten, H. J. W. (1994b). *Zeitschrift fur Physik C*, **64**, 169–176.
Luest, D. and Theisen, S. (1989). *Lectures in String Theory* (Lecture Notes in Physics), *Vol. 346*, Springer Verlag, Berlin.
Maharana, J. (1983). *Physics Letters B* **128**, 411.
Maharana, J. (2000, April). Recent developements in String Theory. In *Quantum Field Theory*, A. N. Mitra, ed., pp. 611–664 (hep-th/9911200).
Mukhi, S. (1997). Recent developements in String Theory: A brief review for particle physicists. (hep-ph/9710470 (TIFR/TH/97-55)).
Schmidhuber, C. (1996). *Nuclear Physics B* **467**, 146–158.
Senjanovic, P. (1976). *Annals of Physics (NY)* **100**, 227–261.
Sheikh-Jabbari, M. M. and Shirzad, A. (1999). Boundary conditions as Dirac constraints. (hep-th/9907055).
Tseytlin, A. A. (1996). *Nuclear Physics B* **469**, 51–57.